27. $\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx$

$u=x^2+2x$

$du=(2x+2)\,dx$

$du=2(x+1)\,dx$

$\frac{du}{2(x+1)}=dx$

$\int \frac{\cancel{x+1}}{\sqrt{u}}\,\frac{du}{2\cancel{(x+1)}}$

$\frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$\frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right)=\sqrt{u}=\sqrt{x^2+2x}$

$\lim_{b\to 0}\sqrt{x^2+2x}\,\Big|_b^1 = \lim_{b\to 0}\sqrt{3}-\sqrt{b^2+2b}=\sqrt{3}$

Jan 24-9:41 AM

8.4b Improper Integrals

Comparison Test

hard to integrate   easy & converge

$0 \le \int_a^\infty f(x)\,dx \le \int_a^\infty g(x)\,dx$

must converge

$\int_a^\infty f(x)\,dx \ge \int_a^\infty g(x)\,dx$

hard   easy & diverge

must diverge

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$\int_1^\infty e^{-x^2}\,dx \le \int_1^\infty e^{-x}\,dx$

converges also

we hope this converges. it does

$\lim_{b\to\infty}\left(-e^{-x}\right|_1^b = \lim_{b\to\infty}\left(-e^{-b}--e^{-1}\right)=\frac{1}{e}$

0

$-\frac{1}{e^b}$

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Does the integral converge or diverge?

$\int_1^\infty \frac{dx}{x^5+1} \le \int_1^\infty \frac{dx}{x^5}$

converges by comparison

we hope it converges. it does!!!

$\lim_{b\to\infty}\int_1^b x^{-5}\,dx$

$\lim_{b\to\infty}\left(\frac{x^{-4}}{-4}\right|_1^b = \lim_{b\to\infty}\left(\frac{b^{-4}}{-4}-\frac{1}{-4}\right)=\frac{1}{4}$

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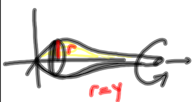
Find the volume of the solid obtained by revolving the curve about the x-axis $y=xe^{-x}, 0\le x<\infty$



$r=y$

$\int_a^b \pi r^2\,dx$ $\int_0^\infty \pi\left(xe^{-x}\right)^2\,dx$

$\pi\int_0^\infty x^2e^{-2x}\,dx$

x^2	+	e^{-2x}
$2x$	−	$-\frac{1}{2}e^{-2x}$
2	+	$\frac{1}{4}e^{-2x}$
0		$-\frac{1}{8}e^{-2x}$

$\lim_{b\to\infty}\pi\left[-\frac{x^2}{2}e^{-2x}-\frac{2x}{4}e^{-2x}-\frac{2}{8}e^{-2x}\right|_0^b$

$\lim_{b\to\infty}\pi\left[-\frac{b^2}{2}e^{-2b}-\frac{2b}{4}e^{-2b}-\frac{2}{8}e^{-2b}-\left(0-0-\frac{2}{8}\right)\right]$

0   0   0

$\frac{\pi}{4}$

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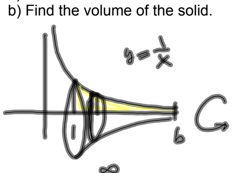
Gabriel's Horn

Consider the region R in the first quadrant bounded above by y=1/x and on the left by x=1. The region is revolved around the x-axis.

a) Show that R has infinite area.

b) Find the volume of the solid.



a) $\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty}\left(\ln x\right|_1^b = \lim_{b\to\infty}\ln b-\ln 1 = \infty$

b) $\int_1^\infty \pi\frac{1}{x^2}\,dx = \int_1^\infty \pi x^{-2}\,dx$

$\lim_{b\to\infty}\left(\frac{\pi x^{-1}}{-1}\right|_1^b = \lim_{b\to\infty}\pi\left(\frac{1}{-b}-\frac{1}{-1}\right)$

0

$=\pi$

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